A PAPER CLOCK MODEL FOR THE CESIUM CLOCK ENSEMBLE OF TL

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Abstract

A paper clock model based on limited numbers of atomic clocks and a typical time interval counter was developed. The aim of this work is to use a small ensemble of cesium clocks and a traditional measurement system to generate a time scale keeping both short-term and long-term accuracy. We removed each clock's drift, then weighted the residual fluctuations. The weight function is set to be proportional to the inverse exponential of the index of each clock's frequency deviation. We find the resultant paper clock to be much more stable and accurate than any contributing clock. The paper clock can keep a ± 50 ns phase difference (compared with UTC) without any calibration, and the Allan deviation of this paper clock is about 1.0×10^{-13} (τ = 10 minutes, compared with hydrogen maser). Here, a typical Allan deviation of the contributing cesium clock is $2.0\sim3.0\times10^{-13}$ in our measurement system. Finally, we use this paper clock to synchronize a hydrogen maser via a phase-lock mechanism. A virtual test showed the phase difference between an adjusted hydrogen maser and the paper clock could be kept within 1 nanosecond without any prediction algorithm.

INTRODUCTION

To develop a stable and accurate time scale is always an endless target for all time and frequency laboratories. TL owns nine cesium clocks (Agilent 5071a) and two hydrogen masers (Kvarz CH1-75); we tried to use this ensemble of clocks to generate a time scale keeping both short-term and long-term accuracy. This time scale would be a reference to synchronize a hydrogen maser via a micro-phasestepper and then generate the new standard time scale of TL. We expect that the short-term stability of the final output would follow the hydrogen maser and long-term accuracy would follow the reference paper clock.

Based on the analysis of clock data, we assumed that the phase difference between any cesium clock and an ideal clock is almost unchanged at several nanoseconds over tens of days if any drift is removed, so that we can average the residual fluctuations of different clocks, and the resultant paper clock would be more stable and accurate than any contributing clock. The other benefit of the drift-removal procedure is that the drift rate of paper clock will not change suddenly when we add a clock to or withdraw one from the ensemble. The weight of each clock is set to be proportional to the inverse exponential of the index of each clock's frequency deviation. We don't need to set any upper limit for the weight function because the inverse exponential has an upper limit itself.

We also developed a mechanism for using this paper clock to synchronize a hydrogen maser via a micro-phasestepper. We compared the 1 PPS phase difference between the paper clock and a hydrogen maser, and adjusted the frequency offset by a fixed amount if the phase of hydrogen maser was advanced or retarded with respect to the paper clock. A simulated result showed that the phase difference between adjusted clock and paper clock can be kept to about 1 ns. Using this mechanism, we can synchronize this paper clock and a stable real clock without any prediction algorithm. This is helpful when we set up more than one backup time scale system; the primary and backup systems will be kept in phase automatically.

CLOCK DATA ANALYSIS

Before we started to develop our algorithm, we had to check the initial character of our cesium clocks. From the BIPM's monthly reports, we found that the drift rate of all our cesium clocks was not a constant (Figure 1). The variation of each clock's drift rate is about 3~10 ns/day over their lifetime. Figure 2 shows the phase difference between UTC and each cesium clock; we notice that they are not linear in the long-term, but are stable during a short period. Figure 4 illustrates that we can have the best stability when the average time is 30 days (CS160, CS300, CS1012, and CS1498) or 60 days (CS1500, CS1712).

Based on the above analysis, we assume the 30~60 day drift rate will not change too much between the first and the second 30~60 day period; that is, we can use the drift rate 30~60 days ago to be the drift rate of the next 30~60 days. Another assumption is that if the drifts of the clocks are removed, the residual fluctuations will be reduced when we sum and average each phase difference of the clocks. The assumption is based on the independence of each cesium clock. The independence of each clock can be kept if we have a stable operating environment.

ALGORITHM AND WEIGHTING PROCEDURE

We first test the equal-weighted average phase difference, UTC-Clocks. The ensemble time scale will be

$$Ens(t) = TL(t) + \frac{1}{N}\sum_{i=1}^{N} x_i(t) \quad \ldots \ (1)$$

Here, we denote $x_i(t) = UTC(TL) - clock_i$, the phase difference between UTC (TL) and each clock; $Ens(t)$ is the UTC – ensemble clock at time t; and $TL(t) = UTC(t) - UTC(TL)(t)$ is the phase difference between UTC and UTC (TL) at time t.

Line 1 of Figure 3 shows the average of phase difference of each clock; the best paper clock can keep ±30 ns accuracy over 500 days, but we have to know the drift rate of this ensemble in advance. When we add one clock to ensemble, the drift of the ensemble will change suddenly (Line 2 of Figure 3). If we remove each clock's drift rate every certain period and then average them, the modified equation becomes:

$$Ens(t) = TL(t) + \frac{1}{N}\sum_{i=1}^{N}[x_i(t) - (t-\Delta t)\cdot d_i(t-\Delta t)] \quad \ldots\ldots (2)$$

Here, the $d_i\,(t\text{-}\Delta\, t)$ denotes the drift rate of $clock_i$ at the period $t\ \text{-}\Delta\ t$ to t, (compared with $Ens\ (t\text{-}\Delta\ t)$). Figure 4 is the result of equation (2); it's much better than the result of equation (1). The result retains good accuracy without any calibration (Line 3 of Figure 3).

All time scale algorithms have a weighting procedure. Giving each clock a weight can filter out unreasonable or erroneous data and be helpful for short-term stability. We expect that the weight will approach zero when the clock is very unstable and approach an upper limit if it is very stable. So we consider an inverse exponential function. We set the weight of each clock to be proportional to the inverse exponential of the index of each clock's Allan deviation. We didn't set any upper limit of weight because the inverse exponential has an upper limit itself.

$$Ens\ (t) = TL\ (t) + \frac{1}{N}\sum_{i=1}^{N} w_i(t)\cdot[x_i(t) - (t-\Delta t)\cdot d_i(t-\Delta t)] \ldots\ldots (3)$$

where $w_i(t) = \dfrac{e^{-b\sigma_i^2(t-\Delta t)}}{\sum_{i=1}^{N} e^{-b\sigma_i^2(t-\Delta t)}}$ and $\sigma_i\ (t\text{-}\Delta\ t)$ is the Allan deviation between hydrogen maser and $clock_i$ during the period $(t\text{-}\Delta\ t)$ to t. Coefficient b can control the behavior of the weight function: large b makes the weight function like an inverse square weight, and small b makes the weight function like an equal weight. Here, we set $b = 0.3\times 10^{26}$.

We use clock data from MJD 52499 to MJD 52850 and test the interval ($\Delta\ t$) from 5 to 60 days. All results could be kept within ± 50 ns when compared with UTC (Figure 5). The interval of 60 days may have the best long-term stability, but has no significant difference compared with the other intervals. It would be better if we could analyze more than 3 years of data so that we can verify a more long-term result. We also tested adding a clock to the ensemble: CS474 was added to this ensemble at MJD 52650. Figure 6 shows the influence of adding a clock to this ensemble: the long-term drift rate did not change too much (about 0.1~0.2 ns/day over 200 days); even the drift rate of CS474 was about 22.2 ns/day (Figure 6).

Another test is the comparison of different kinds of weighting procedures; we calculated different weighting procedures and compared their stability with that of a hydrogen maser. We tested:

equal weighting, $w_i(t) = \dfrac{1}{N}$

inverse exponential weighting, $w_i(t) = \dfrac{e^{-b\sigma_i^2}}{\sum_{i=1}^{N} e^{-b\sigma_i^2}}$

and inverse square weighting, $w_i(t) = \dfrac{1/\sigma_i^2}{\sum_{i=1}^{N} 1/\sigma_i^2}$.

As we expected, all results had a much better stability than that of the contributing clocks (Figure 7). The Allan deviation of the paper clock is about 1×10^{-13} (σ = 300 seconds), and that of the contributing cesium clocks is about $2\sim3\times10^{-13}$. The weighting procedure did benefit short-term stability, but to a very small extent, and we cannot find any significant difference between inverse exponential weighting and inverse square weighting (Figure 8).

PHASE-LOCK MECHANISM

Since an atomic clock (cesium or hydrogen maser) will not change its drift rate very rapidly in a short period, one idea is that we can lock the phase between an atomic clock and this paper clock just like a phase-lock loop. A phase-lock mechanism was developed to lock this paper clock and an atomic clock via a simulated micro-phasestepper. We compared the phase difference between the paper clock and an atomic clock every certain interval, adding or reducing the frequency offset of a virtual micro-phasestepper with a fixed amount if the phase of the synchronized atomic clock is advanced or retarded with respect to the paper clock; the rule can be stated below:

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 if (phase difference >  a ns) {$drift = $drift + b ns/day}
if (phase difference < -a ns) {$drift = $drift - b ns/day} ................(5)
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We tested two cesium clocks and one hydrogen maser (CS1712, CS1498, and HM76052). We noticed that the coefficient b and checking interval must be correlated to achieve the best synchronous performance; a small coefficient b is related to frequent checking and better short-term accuracy. The coefficient a is a constraint, depending on how accurate one wants to synchronize. There exists a limitation on coefficient a: the short-term stability of clocks; if the 1-hour Allan deviation of a clock is 1×10^{-13}, it is about 0.36 ns variation itself, so that we cannot expect to have an accuracy better than 0.36 ns.

Here, we check the phase difference every hour. The coefficients a = 1.0 ns and b = 0.015 ns/day for cesium clocks; a = 0.1 ns and b = 0.02 ns/day for a hydrogen maser. Figure 9 is the result of phase-lock mechanism. As we expect, there are only a few significant phase differences between the paper clock and the synchronized clock (± 2 ns for a cesium clock and ± 1 ns for a hydrogen maser), and the hydrogen maser would have the better accuracy because of its greater stability.

DISCUSSION AND CONCLUSION

There are some advantages of this paper clock model. The first one is: this paper clock just uses a traditional measurement system (which records phase difference by a switch system and a time interval counter) and simple data processing to generate an accurate and stable time scale. The second is: we don't need to care that the drift rate will change rapidly if one clock is withdrawn from or added to the ensemble. The third advantage is: we can use the phase-lock mechanism to synchronize any stable atomic clock without any prediction algorithm. Another one is: the different clocks can be synchronized at the nanosecond level by a single time scale, which means that we can generate more than one backup time scale system and don't need to take care of the phase difference between the primary and backup systems.

ACKNOWLEDGMENTS

I greatly appreciate the help of Mr. Imae, Dr. Hanodo, and Dr. Hosokawa; they gave me much support when I was a guest researcher at CRL, Japan, so that I could analyze the clock data of CRL and get the long-term characteristics of cesium clocks.

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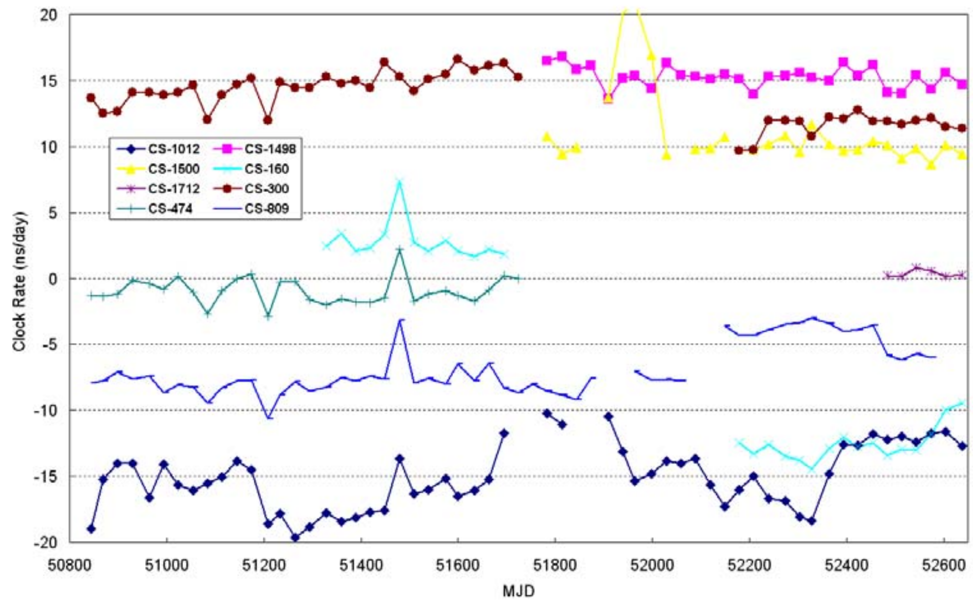


Figure 1. Cesium clocks' drift rates (from BIPM monthly reports).

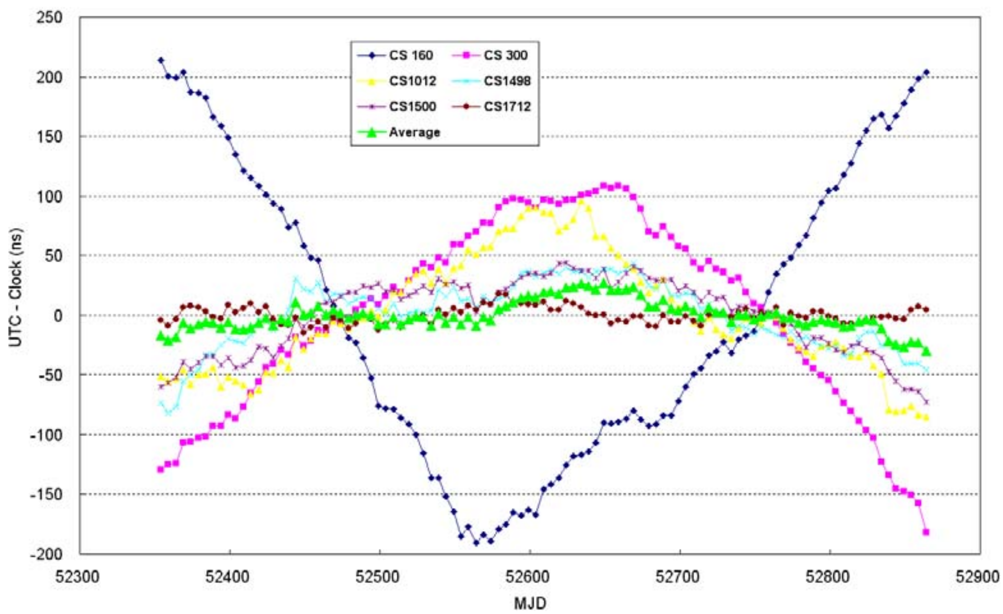


Figure 2. The phase difference between UTC and cesium clocks; all drift of each clock is removed. Notice that no clock is linear.

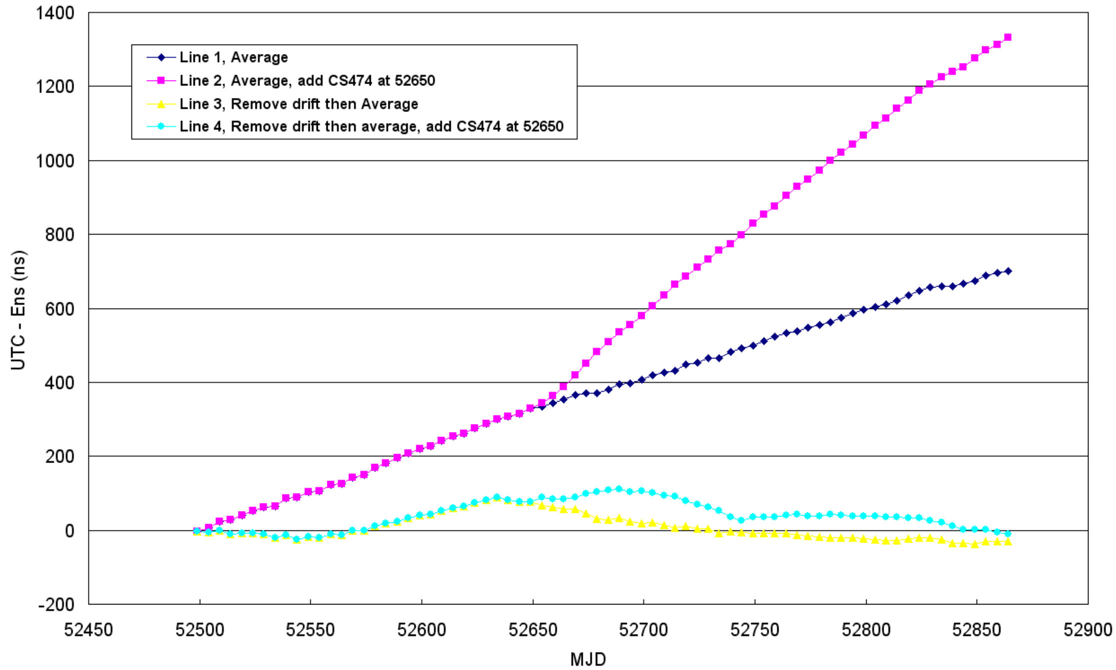


Figure 3. The phase difference between UTC and some test paper clocks.

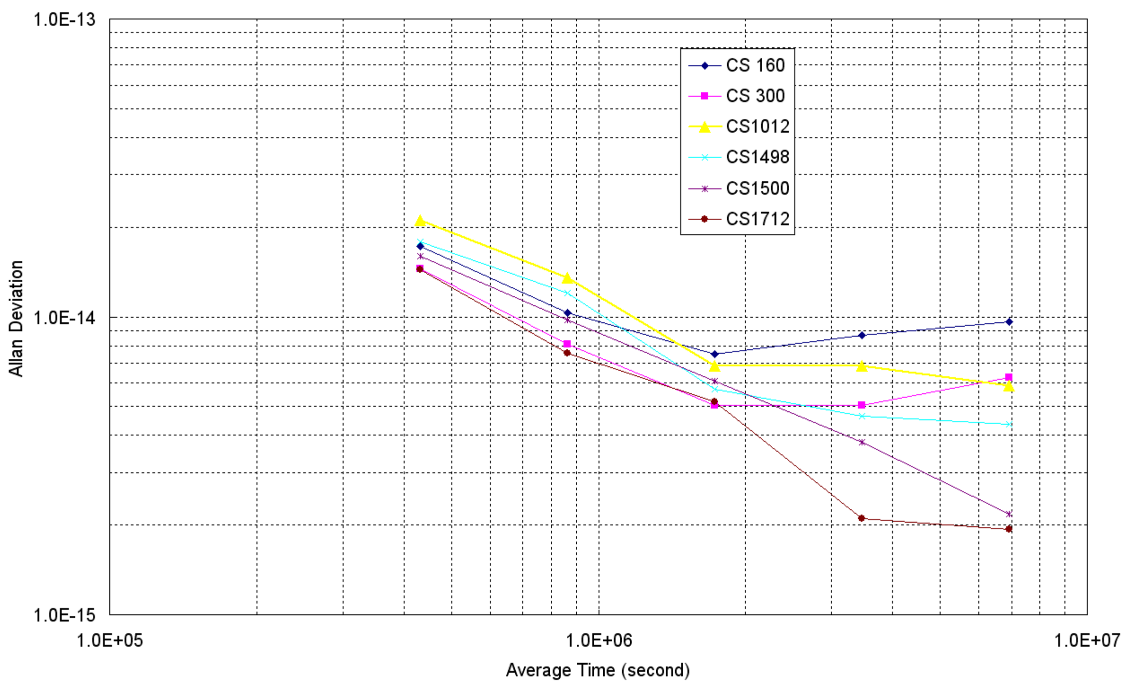


Figure 4. Allan Deviation of cesium clocks vs. UTC.

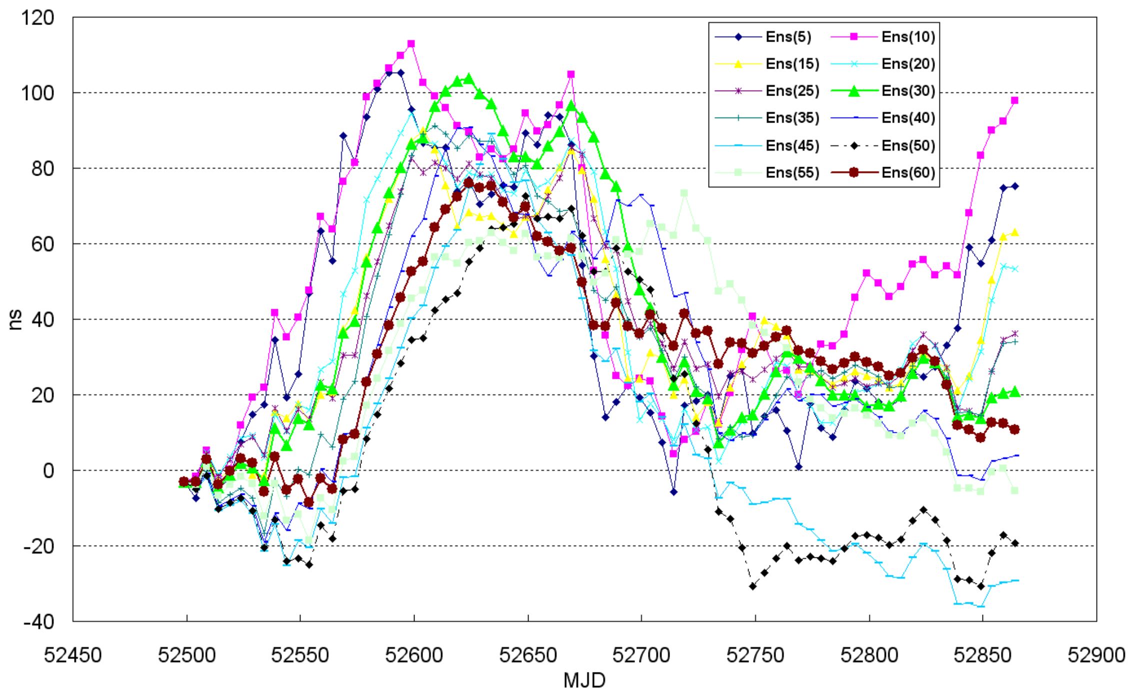


Figure 5. The test of different drift removal intervals, from 5 days to 60 days.

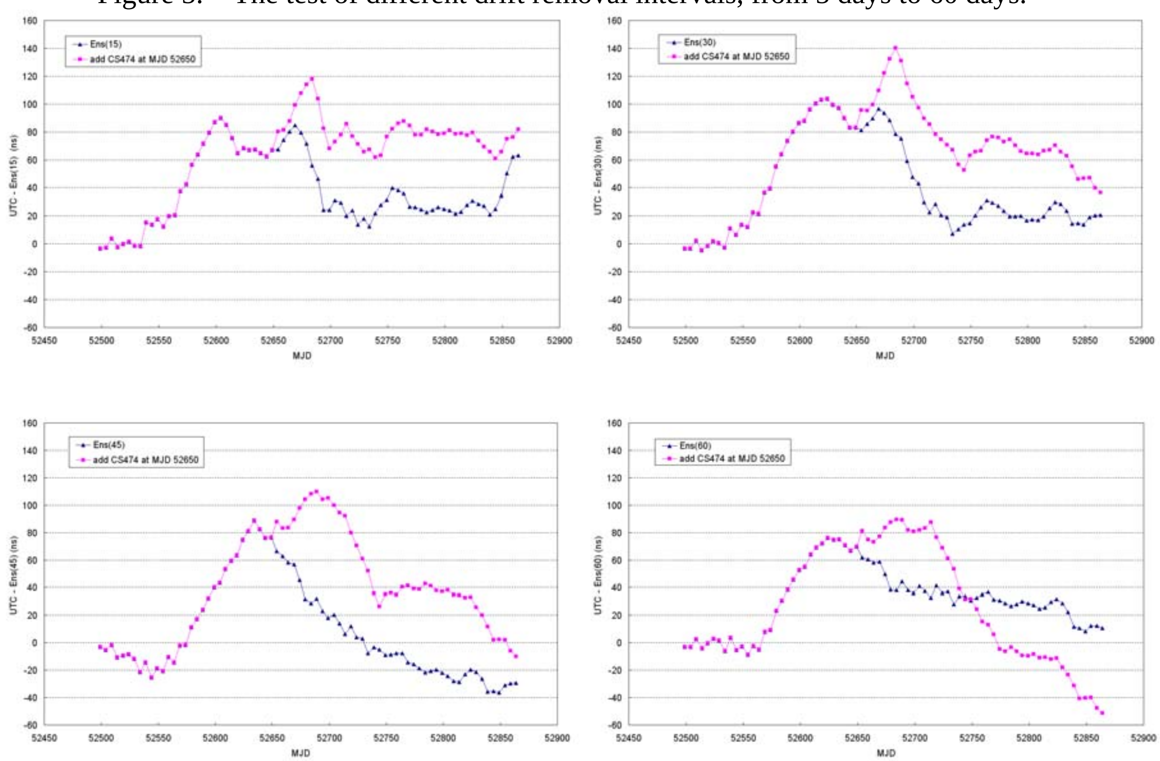


Figure 6. The test of adding one clock to the ensemble (CS474, MJD 52650). Drift removal intervals for 15, 30, 45, and 60 days are tested.

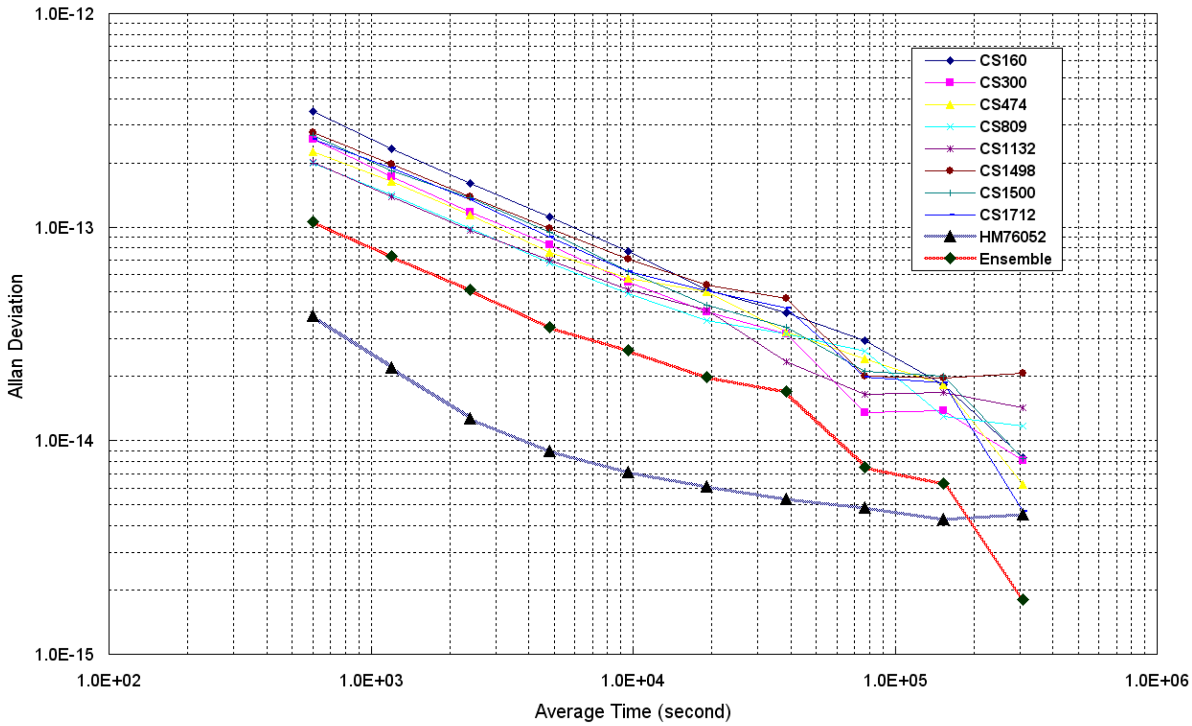
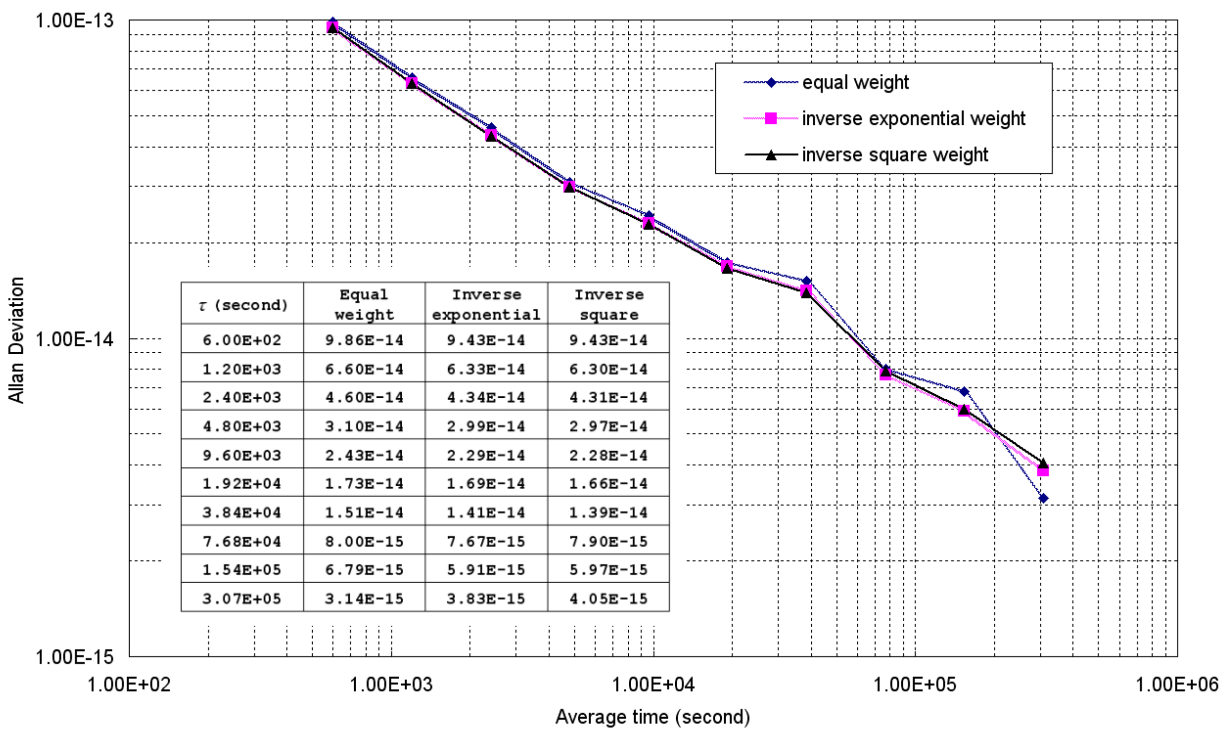


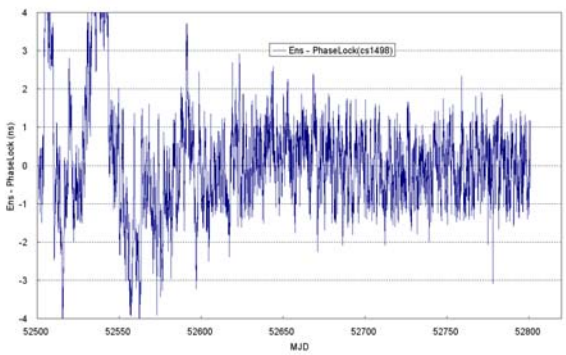
Figure 7. Stability of cesium clocks and the paper clock vs. a hydrogen maser.

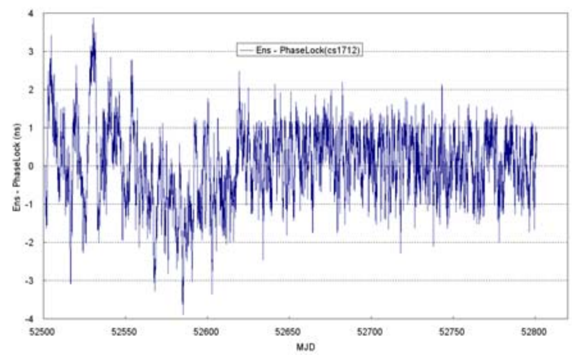


τ (second)	Equal weight	Inverse exponential	Inverse square
6.00E+02	9.86E-14	9.43E-14	9.43E-14
1.20E+03	6.60E-14	6.33E-14	6.30E-14
2.40E+03	4.60E-14	4.34E-14	4.31E-14
4.80E+03	3.10E-14	2.99E-14	2.97E-14
9.60E+03	2.43E-14	2.29E-14	2.28E-14
1.92E+04	1.73E-14	1.69E-14	1.66E-14
3.84E+04	1.51E-14	1.41E-14	1.39E-14
7.68E+04	8.00E-15	7.67E-15	7.90E-15
1.54E+05	6.79E-15	5.91E-15	5.97E-15
3.07E+05	3.14E-15	3.83E-15	4.05E-15

Figure 8. The comparison of three kinds of weighting procedures: equal weighting, inverse exponential

weighting, and inverse square weighting. The equal weighting procedure is slightly less stable than the other two, but not significantly.





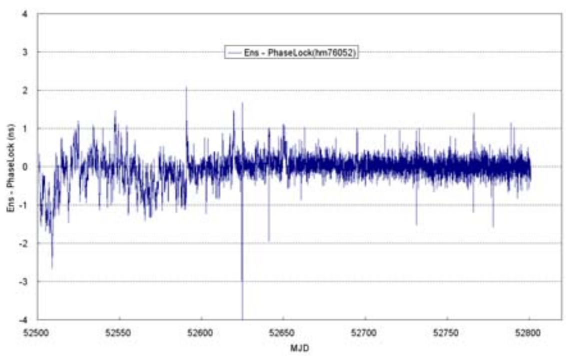


Figure 9. Results of a phase-lock mechanism: two cesium clocks (CS1498 and CS1712) and a hydrogen maser (HM76052) are chosen to be the reference clock.

QUESTIONS AND ANSWERS

DAVE HOWE (National Institute of Standards and Technology): I just have a few comments. One is that ordinarily, when you talk about drift in a cesium standard, I just want to be clear that what you are really looking at is piece-wise drift estimates.

The second thing is that your drift estimator is a three-point estimator, and so I just want to caution you that the results in the composite will be optimistic, because the three-point estimator happens to also be the point estimator for the Allan variance. When you remove it, the Allan variance will have a zero result at that averaging time. When you look at the composites, be very careful about thinking that it is as good as the claim of 0.2 nanoseconds per day in terms of stability on the long term because of the way you are removing drift piece-wise.

The last comment is that ordinarily you can weight clocks of equal types of noise. I think it would be useful to look at the addition and deletion of clocks which may have differing mix of noise. That would be a good test.

SHINN-YAN LIN: Yes, I agree with you. But at the beginning, we were looking for a very simple clock model. At that stage, we did not think about so many things.

One of our points is that we actually used seven cesiums. The estimation of Clock One has some error. Clock Two has some error. But the error might be averaged, over several hundred days. We can calibrate the ensemble with UTC. That is our point.